**Specialist Mathematics: Unit 3**

**Investigation 1 – Complex Plane Dynamics**

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Due: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Part 1: Take Home Component**

Complete this Take Home Component on file paper showing all working out and reasoning. Use of CAS calculator to aid calculation is assumed. On completion of Part 1 there will be a Validation Task (Part 2). For Part 2, CAS calculators will be allowed but no other notes will be permitted.

**Spiral in or out?**

1. What happens when a complex number is multiplied by itself repeatedly?
2. Consider the complex number .

List each complex number in the sequence .

Plot each point on an Argand diagram and connect with straight lines.

Describe the resulting pattern.

1. Repeat this process for the values of below. That is, list the sequence and plot each set of points on an Argand diagram. Connect the points with straight lines and describe the resulting pattern.
2. Which complex numbers spiral inwards, which spiral outwards and which do not spiral at all? Generalise your findings. Test your conjecture by choosing several other complex numbers and repeatedly multiplying them by themselves.

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Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Part 2: Validation Test Marks: /55**

Write your responses in the space provided.

**SECTION A: 25 Marks**

**1. (4, 2, 1, 1, 1, 2 =** **11 marks**)

(a) Complete the table and plot the first 5 points generated by the sequence 1, z, z2, z3, z4, ....

where  and connect them with straight lines. (4 marks)

|  |  |  |
| --- | --- | --- |
| **Cartesian** | **Polar** |  |
| Z0 = 1 | Z0 = |  |
| Z = | Z = |
| Z2 = | Z2 = |
| Z3 = | Z3 = |
| Z4 = | Z4 = |
|  |  |

(b) Explain, using polar form, how the location (modulus and argument) of each point

(excluding the first) may be calculated from the one before it. (2 marks)

(c) How can you determine  if you know ? (1 mark)

(d) How can you determine if you know ? (1 mark)

(e) Why is 1 the logical T0 term in such a sequence? (1 mark)

(f) One of the terms in the sequence will be **w cis (-π)**. State **which term it is** and **give the**

**value of w.** (2 marks)

**2. (3, 1, 1, 1, 2 = 8 marks)**

Consider the sequence 1, z, z2, z3 , z4, .... where z = 0.48 + 0.2i

(a) Give the polar coordinates,  of the first four terms of the sequence. (Give the

argument in radians accurate to 2 d.p. and the modulus accurate to 4 decimal places)

(3 marks)

(b) State the two major differences between the spiral in Q.4 and the spiral that would be

created here. (No need to draw) (1 mark)

(c) How can you determine  if you know ? (1 mark)

(d) How can you determine if you know ? (1 mark)

(e) Find the lowest value of n such that Arg(zn) > 4π. (2 marks)

**3. (1, 1 = 2 marks)**

(a) Determine a formula to calculate  given you know . (1 mark)

(b) Determine a formula to calculate given you know . (1 mark)

**4. (4 marks)**

A spiral of the sequence 1, z, z2, z3 ... z10 finishes with [243,0]. That is .

Using exact values, find  and give the smallest possible positive value for Arg(z).

**SECTION B: 30 Marks**

**5.** **(2, 4, 4, 4, 1, 2 = 17 marks)**

Consider the sequence generated by multiplying a complex number, , by itself:

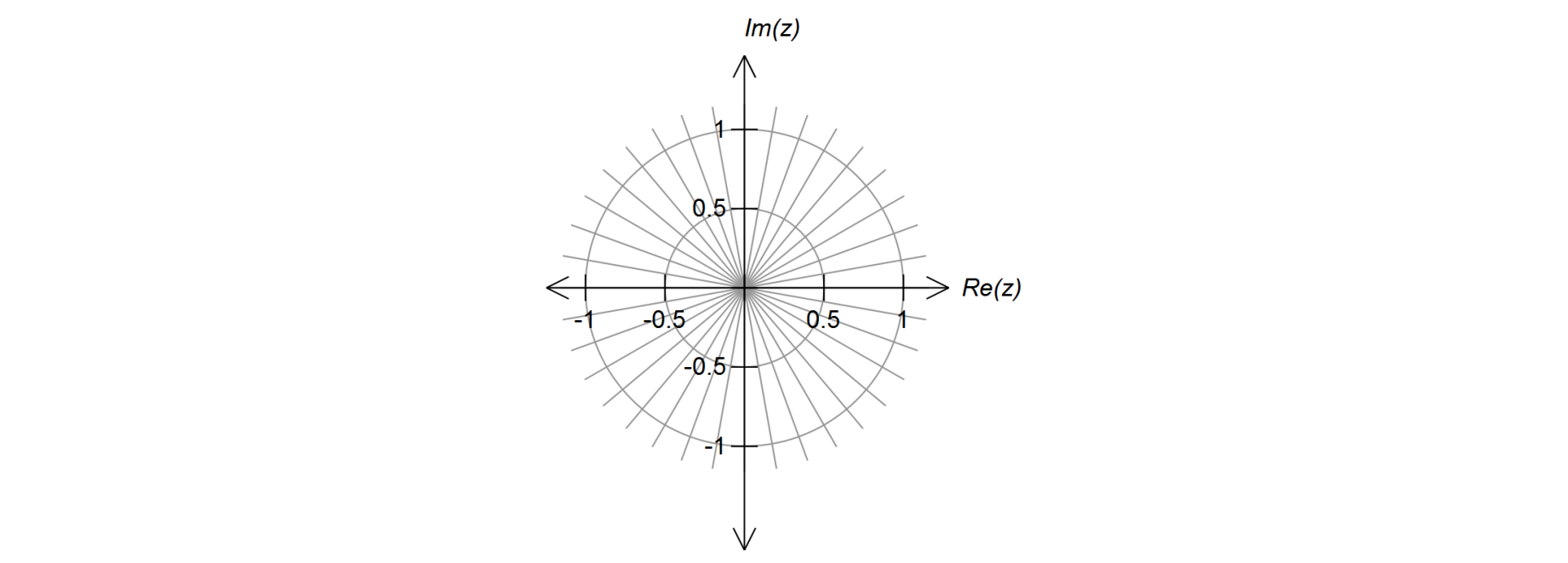
In the Take Home Component you should have discovered that if the modulus of is 1, the points generated neither spiral in or out. In this question you will investigate the role of the argument of in these sequences.

1. List and plot the points generated by the sequence for the following complex numbers and connect them with straight lines. You need to continue the sequence for as many terms as necessary to form a closed shape.

For the next two parts

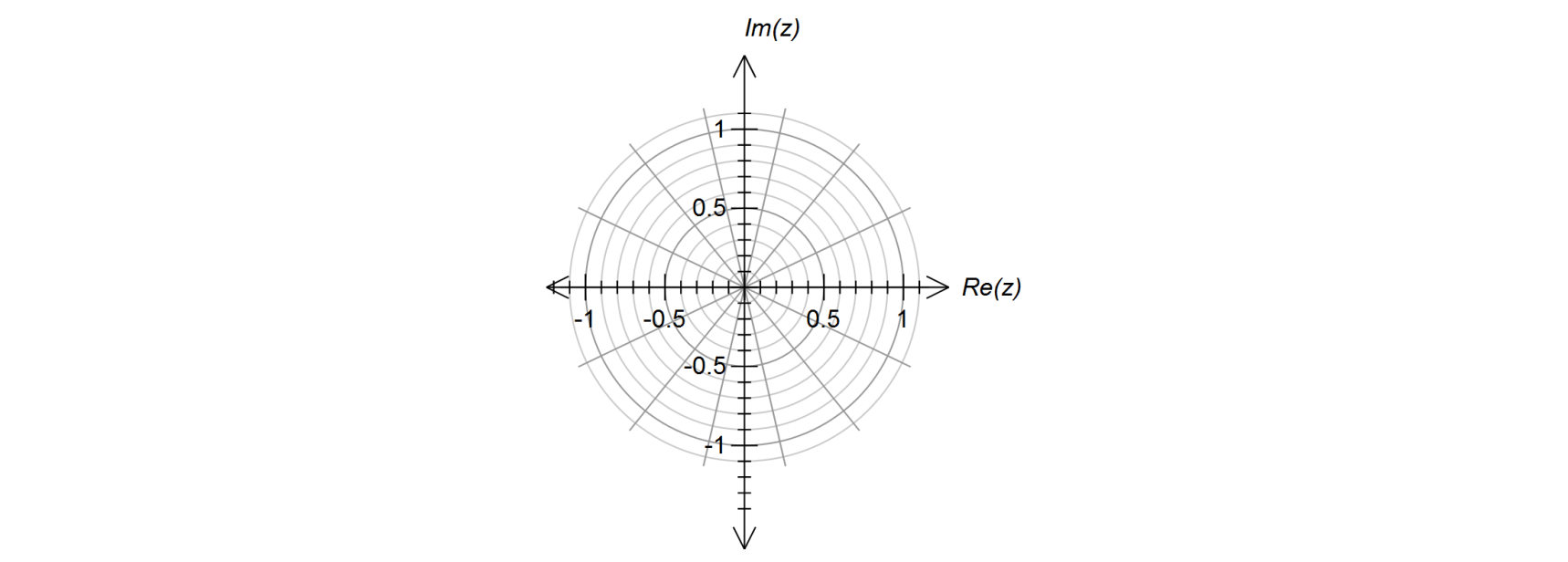
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* 1.  (4 marks)

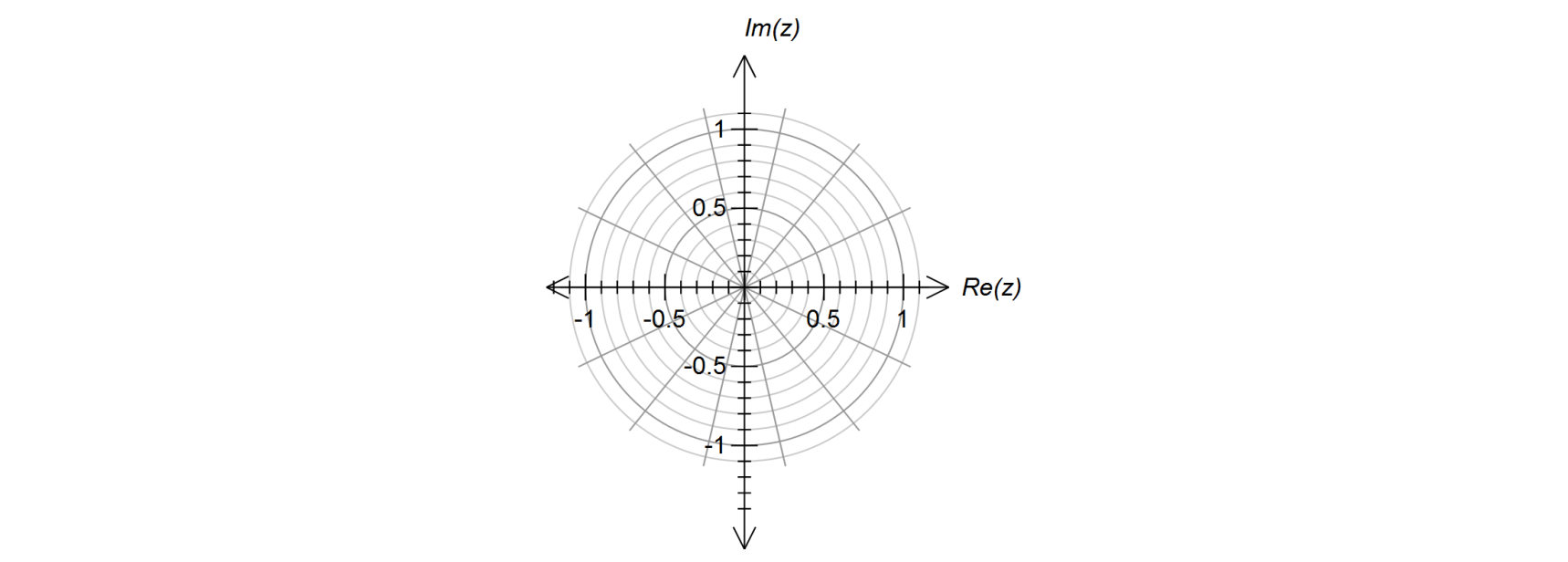
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For the next two parts

* 1.  (4 marks)

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* 1. (4 marks)



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1. Generalise your results. What values of will produce closed polygons in one revolution? (1 mark)
2. All of the above sequences have rotated anticlockwise. Determine a complex number that would produce a closed polygon by rotating clockwise. State the type of polygon that would be formed. It should be a different polygon to those drawn above. (2 marks)

**6.** **(2, 2, 1, 1 = 6 marks)**

An equilateral triangle, drawn on an Argand diagram and centred at the origin has one of its vertices at .

1. Give the polar form of the complex numbers that define the other two vertices.

Give . (2 marks)

1. Convert your answers above to Cartesian form (2 marks)
2. Raise each of the complex numbers that form the vertices of the triangle to the power of 3. (1 mark)
3. Hence state the complex solutions to (1 mark)

**7.** **(1, 5, 1 = 7 marks)**

is one of the solutions to the equation , where is a complex number.

1. Find . (1 mark)
2. is one vertex of a regular pentagon. Give the polar form of the complex numbers that define the five vertices of the pentagon. Give . (5 marks)
3. Hence solve , in polar form. (1 mark)